Worksheet: Modelling using Quadratic Functions

Short Answer

1. A quadratic function has these characteristics:
   $x = 1$ is the equation for the axis of symmetry.
   $x = -1$ is an x-intercept.
   $y = -4$ is the minimum value.
   Determine the y-intercept of this parabola.

2. Given $f(x) = -3x^2 + 6x + 7$, determine the equation of the inverse. Explain how you found your answer.

3. An integer is seven more than another integer. Twice the larger integer is one less than the square of the smaller integer. Find the two integers.

4. Is the function shown linear or quadratic? Explain your answer.

x	y
–1	–10
0	–20
1	–26
2	–28
3	–26

5. At a baseball game, workers toss T-shirts to spectators in the stands out of a sling-shot. The height of a T-shirt is modelled by the function $h(t) = -5t^2 + 20t + 1$ where $h(t)$ is height in metres and t is the time in seconds after the toss. What is the maximum height of the T-shirt if it is not caught? How much time does it take the T-shirt to reach maximum height?

6. An ice cream company varies the prices of its pint containers to maximize profit. The function $P(x) = -80(x-3)^2 + 150$ models the company’s profits in thousands of dollars, where x is the price of a pint of ice cream in dollars. At what price will the company receive maximum profits? How much profit will the company earn?

7. Determine the maximum value for the function $f(x) = -x^2 - 4x - 32$. Explain how you found your answer.

8. Christine has a 180-cm strip of wood to make a frame. Determine a function to represent the area of the frame, $f(x)$, based on the length of the frame, x. What is the maximum area Christine can make for the frame?

9. The cost, $c(x)$, in dollars per hour of running a certain fishing boat is modelled by the function $c(x) = 0.9x^2 - 18.1x + 135.1$, where x is the speed in kilometres per hour. At what approximate speed should the boat travel to achieve minimum cost?

10. The demand function for a new perfume is $p(x) = -2x + 36$ where $p(x)$ represents the selling price, in thousands of dollars, and x is the number of bottles sold, in thousands. Determine the revenue function and the maximum revenue.

11. The cost function for a container company is $C(x) = 10x + 30$ and the revenue function is $R(x) = -x^2 + 24x$, where x is the number of containers sold, in thousands. Determine the profit function for the number of containers sold. Then determine the number of containers sold that maximizes profit.

12. Sharon holds a soccer ball and punts it with her foot. The function $h(t) = -5t^2 + 20t + 1$ models the height of the ball in metres at time t seconds after contact. There is a wall in front of Sharon with a window 25 m high. Will the ball hit the window? Explain your answer.

13. The cost, $c(x)$, in dollars per hour of running a certain steamboat is modelled by the function $c(x) = 1.7x^2 - 13.6x + 166.4$, where x is the speed in kilometres per hour. At what approximate speed should the boat travel to achieve minimum cost?

14. The cost function for a clock factory is $C(x) = 7x + 27$ and the revenue function is $R(x) = -4x^2 + 39x$, where x is the number of clocks sold, in thousands. Determine the profit function for the number of clocks sold. Then determine the number of clocks sold that maximizes profit.

15. Travis and Laura are rock climbing. Travis throws a spike to Laura. The function $h(t) = -5t^2 + 20t + 110$ models the height of the spike in metres above the ground at time t. Laura is 135 m above the ground. Did Travis' throw reach Laura? Explain your answer.

Problem

16. Wayne threw a ball over a 3-metre wall. The ball just cleared the wall without any additional space. The ball landed 9 m from the wall.
a) Using the wall as the axis of symmetry, write a function in vertex form that approximates the path of the ball. (Let the origin be where the wall meets the ground.)
b) Describe how you found your function.
c) Graph your function.
d) State the domain and range.

17. The function $E(t) = 3t^2 - 48t + 900$ models the production expenses for a bicycle company in thousands of dollars where t represents time in years.
a) Write the function in vertex form.
b) Determine the model that describes time in terms of expenses.
c) Graph your relation for domain $\{t \in \mathbf{R} \mid t \geq 0\}$.
d) Determine how many years have passed once production expenses reached $900 000.

18. Determine the number of zeros of the function $f(x) = 7 - (x - 5)(4x - 2)$ without solving the related quadratic equation or graphing. Explain your thinking.

19. A highway tunnel has a shape that can be modelled by the equation of a parabola. The tunnel is 18 m wide and the height of the tunnel 16 m from the edge is 5 m.
a) Determine the equation of the parabola.
b) Sketch a graph of your parabola.
c) Can a truck that is 8 m tall and 4 m wide pass through the tunnel? Justify your decision.

20. A quadratic function is defined by $f(x) = -3.7x^2 + 6.8x + 4.2$. A linear function is defined by $g(x) = -0.5x + k$.
a) Determine the value of k so that the line intersects the parabola at exactly one point. Write your answer to the nearest hundredth.
b) Sketch a graph of your answer.
c) Determine the values of k so that the line intersects the parabola at two points.

d) Determine the values of *k* so that the line never intersects the parabola.

Worksheet: Modelling using Quadratic Functions
Answer Section

SHORT ANSWER

1. ANS:
(0, –3)

PTS: 1 REF: Thinking OBJ: 3.1 - Properties of Quadratic Functions

2. ANS:
$f^{-1} = 1 \pm \sqrt{\dfrac{-x+10}{3}}$; First I wrote the function in vertex form: $f(x) = -3(x-1)^2 + 10$. I switched *x* and *y* in the function and solved for *y*. First I subtracted 10 from both sides, then I divided both sides by –3, took the square root of both sides, and finally I added 1 to both sides.

PTS: 1 REF: Communication
OBJ: 3.3 - The Inverse of a Quadratic Function

3. ANS:
5 and 12; –3 and 4

PTS: 1 REF: Thinking OBJ: 3.8 - Linear-Quadratic Systems

4. ANS:
The function is quadratic. I graphed the points on a coordinate grid and they formed a parabola.

PTS: 1 REF: Communication
OBJ: 3.1 - Properties of Quadratic Functions

5. ANS:
The maximum height is 21 m. It reaches 21 m at 2 seconds.

PTS: 1 REF: Thinking OBJ: 3.1 - Properties of Quadratic Functions

6. ANS:
$3; $150 000

PTS: 1 REF: Thinking OBJ: 3.1 - Properties of Quadratic Functions

7. ANS:
The maximum value is –28. I converted the function to vertex form, $f(x) = -(x+2)^2 - 28$, and found that the vertex is (–2, –28). Since the parabola opens down, –28 is the maximum. (Other methods include finding the axis of symmetry and sketching a graph of the parabola.)

PTS: 1 REF: Communication
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

8. ANS:
$f(x) = -x^2 + 90x$; 2025 cm^2

PTS: 1 REF: Thinking
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

9. ANS:
about 10 km/h

PTS: 1 REF: Application
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

10. ANS:
$R(x) = -2x^2 + 36x$; $R(x)$ = \$162 000

PTS: 1 REF: Application
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

11. ANS:
$P(x) = -x^2 + 14x - 30$; $x = 7000$

PTS: 1 REF: Thinking
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

12. ANS:
No, the ball will not hit the window. The maximum height of the ball is 21 m at 2 seconds.

PTS: 1 REF: Communication
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

13. ANS:
4 km/h

PTS: 1 REF: Application
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

14. ANS:
$P(x) = -4x^2 + 32x - 27$; $x = 4000$

PTS: 1 REF: Thinking
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

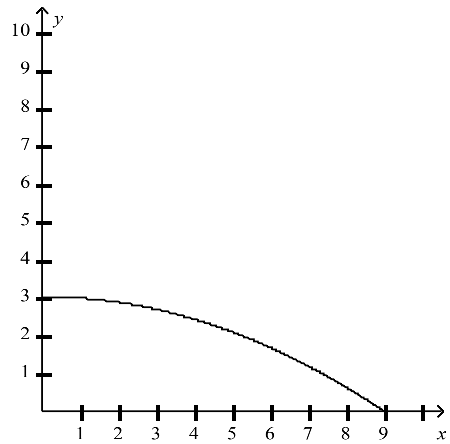
15. ANS:
No, Travis' throw did not reach Laura. The maximum height of the spike is 130 m at 2 seconds.

PTS: 1 REF: Communication
OBJ: 3.2 - Determining Maximum and Minimum Values of a Quadratic Function

PROBLEM

16. ANS:
a) $f(x) = -\frac{1}{27}(x - 0)^2 + 3$

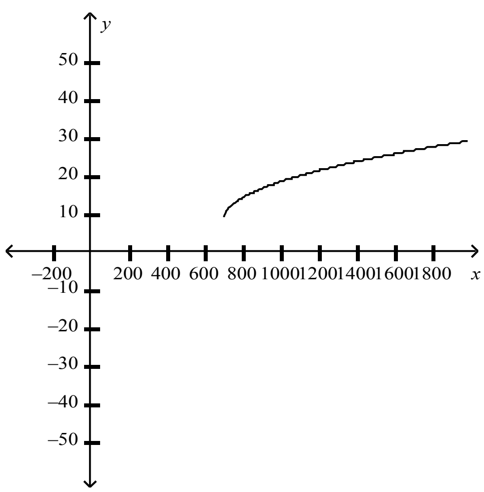
b) Since the top of the wall is the vertex, I substituted (0, 3) into the vertex form to get $f(x) = a(x - 0)^2 + 3$. Since the ball landed 9 m from the wall, I substituted (9, 0) into the function and solved for *a*.



c)
d) Domain = $\{x \in \mathbf{R} \mid 0 \leq x \leq 9\}$, Range = $\{y \in \mathbf{R} \mid 0 \leq y \leq 3\}$

PTS: 1 REF: Communication
OBJ: 3.1 - Properties of Quadratic Functions

17. ANS:
a) $E(t) = 3(t - 8)^2 + 708$
b) $t = 8 \pm \sqrt{\dfrac{E - 708}{3}}$



c)
d) 16 years

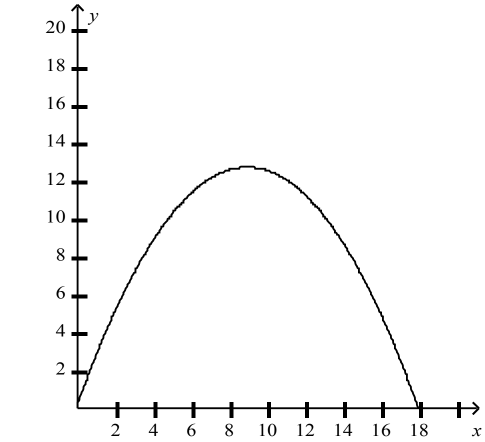
PTS: 1 REF: Thinking OBJ: 3.3 - The Inverse of a Quadratic Function

18. ANS:
There are two zeros. I simplified the function to $f(x) = -4x^2 + 22x - 3$ and calculated the discriminant to get 436. Since the discriminant is greater than zero, the function has two zeros.

PTS: 1 REF: Thinking OBJ: 3.6 - The Zeros of a Quadratic Function

19. ANS:

a) $f(x) = -\frac{5}{32}x(x - 18)$



b)

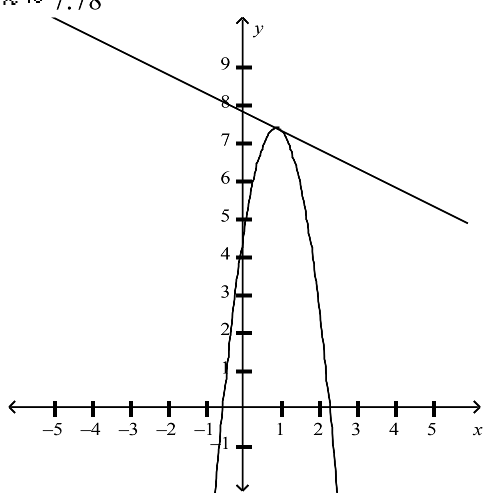
c) Yes, the truck can pass through the tunnel. The height of the tunnel is greater than 8 m for about 10 m in width, so there is enough room for the truck to pass through.

PTS: 1 REF: Communication

OBJ: 3.7 - Families of Quadratic Functions

20. ANS:

a) $k \approx 7.78$



b)

c) $k < 7.78$

d) $k > 7.78$

PTS: 1 REF: Thinking OBJ: 3.8 - Linear-Quadratic Systems